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Instabilities in the slit-jet flow field

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Three modes of instabilities in the slit-jet flow field are recognized. Additional evidence for the universality of the Strouhal number for the second mode $S_T = fw/U_0 \approx 0.43$ and additional information on the wavelength (λ) between, and convection speed u_c of the symmetrically placed, large-scale motions that result from this instability are presented. Specifically $\lambda/w \approx 1.2$ and $u_c/U_0 \approx 0.51$. The third instability mode is initiated at a Reynolds number $U_0 w/\nu$ of approximately 1600; this instability results in a loss of the regular pattern associated with the large-scale motions.

1. Introduction

The slit-jet flow field, and the definition of the symbols used herein, are shown in figure 1; the indicated streamlines are from the potential-flow solution of Birkhoff & Zarantello (1957). With the exception of quite small Reynolds numbers $U_0 w/\nu$, the experimentally observed velocity and pressure distributions are well approximated by this solution since the viscous boundary layers are thinned by the strong acceleration as the fluid approaches the exit plane. The inviscid core flow, which is bounded by thin laminar boundary layers on the inside nozzle surfaces, represents the 'basic state' for the hydrodynamic instabilities in the slit-jet flow field. The present study is an experimental investigation of these natural instabilities; it clarifies and extends the extant knowledge concerning them.

A similar, but not directly related, flow field has been extensively examined by Anderson. These results are reported in a series of publications (see e.g. Anderson 1954). Specifically, Anderson has related the acoustic properties to the hydrodynamic instability properties of a circular orifice flow with a finite lip thickness t. The controlling parameter (in the absence of upstream cavity-resonance effects) is the length ratio t/d. As noted by Anderson: 'Only when the orifice becomes almost a knife blade does the sound disappear'. In contrast, the present investigation is specifically concerned with the condition $t/w \rightarrow 0$.

Two forms of hydrodynamic instability can be readily identified in the slit-jet flow field. The first of these, the 'rolling-up' of the shed boundary layer, has not been directly observed, but its presence is inferred from the closely similar flow studied by Pierce (1964). Specifically, Pierce shows that a curved shear layer‡ is unstable and that the passage frequency β_r of the small (unit) vortex motions scales with the boundary-layer parameters; viz $\beta_r \delta^* / U_\infty$ = constant (δ^* = displacement thickness at separation). The similar roll-up in a non-curved shear layer has been extensively

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‡ The flow studied by Pierce can be described as a 'canoe paddle set normal to the oncoming stream'. The curved shear layer is formed as the stagnation streamline separates from the lateral edge of the paddle.

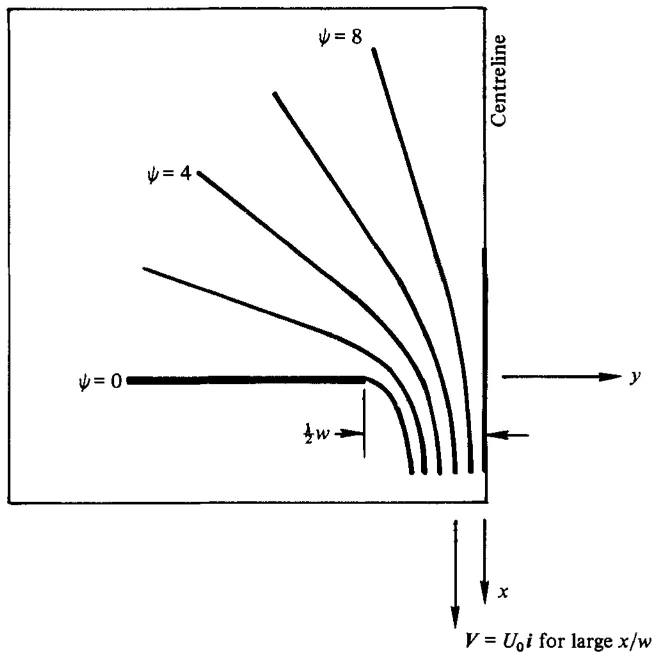

FIGURE 1. The streamline ψ-pattern for the slit-jet flow field. This figure is taken from Birkhoff & Zarantello (1957). The (x, y)-coordinate system is for the present, not the original, study. The reference velocity U_0 is given by $[2\,p_{\text{plenum}}/\rho]^{\frac{1}{2}}$.

studied; Michalke (1972) provides an excellent discussion of the instabilities in the planar problem. This first instability may be identified as a 'shear-layer' instability.

A second instability, and one that represents a dramatically smaller frequency for moderate Reynolds numbers, has been studied by Beavers & Wilson (1970). Their experimental observations are based upon dye streams that were introduced in the boundary layers of the nozzle walls. These dye streams were observed to form symmetric agglomerations of dye ($R_e \gtrsim 200$) which are inferred to represent concentrations of vorticity. It is interesting to note that numerical experiments, by these authors, support this interpretation. The appropriate scales for this second instability are the nozzle width w and the 'inviscid-flow velocity' U_0. The passing frequency f of the 'large' vortex motions may be used to form a Strouhal number $S_T = fw/U_0$. The principal result from the Beavers & Wilson study is that this Strouhal number has a constant value $S_T \approx 0.43$ over the Reynolds-number range: $200 \lesssim U_0 w/n \lesssim 1500$. Hence, unlike the first instability, the second is not dependent upon the boundary-layer lengthscale at separation. Since the flow initiates from a (zero-velocity) plenum, and since the flow was shown to be uninfluenced by the aspect ratio,† it is concluded that the numerical value of $S_T = 0.43$ is a universal value.

It is instructive to consider the relative magnitudes of the passing frequency β_r of the unit vortex motions and the vortex motions from this second instability. An approximate comparison can be formed by using the results from Pierce and by 'equating' the two reference velocities. That is, if U_∞ and U_0 are considered to

† The channel dimensions for the Beavers & Wilson study are given in Foss (1980). Their aspect ratios were $2.94 \leqslant h/w \leqslant 8$.

represent physically equivalent reference velocities, then the ratio of the Strouhal numbers becomes

$$\frac{\beta_r \delta^*/U_\infty}{fw/U_0} = \frac{0.09}{0.43} = 0.21, \quad \frac{\beta_r}{f} \approx 0.21\frac{w}{\delta^*}. \tag{1}$$

Since $w/\delta^* \gtrsim O(10^2)$ for even moderate Reynolds numbers, a large number of 'unit vortex motions' will combine to form the larger vortex motion. This number will vary with Reynolds number, but the passage frequency f and the circulation of these large vortex motions are kinematically determined by U_0 and w.

A third instability is suggested by the text of the Beavers & Wilson paper: 'At higher Reynolds numbers, the number of rings in the regular pattern continues to decrease until at a Reynolds number of about 3000 no regular pattern beyond the first vortex ring can be observed.' This discussion is for a circular orifice; it is inferred that the upper limit for the slit-jet studies† of $U_0 w/\nu \approx 1500$ is from a similar loss of a coherent pattern.

Clark & Kit (1980) present, as a secondary‡ aspect of their paper, $S_T = S_T(R_e, w)$ values. The Strouhal values are independent of the Reynolds number for a given w; however, their data clearly show an unexplained dependence upon the dimensional value w. Foss (1980), in a discussion of their paper, called attention to the inappropriateness of this dependence and suggested that it might represent some form of an apparatus-dependent effect.§ Both of these sharp-edge orifice studies were carried out in water channels whose nominal downstream dimensions were not sufficiently large to preclude the possibility of 'hydrodynamic feedback mechanisms'.

It was concluded that an independent investigation, that would provide not only Strouhal number but convection speed u_c/U_0 and wavelength λ/w information, would be appropriate to determine if the $S_T = 0.43$ result is indeed universal. Furthermore, if the experiment were conducted with air as the working medium and if the receiver downstream of the nozzle were 'unbounded' (i.e. the laboratory), then a possible source of a 'feedback' effect (i.e. a 'small' receiver) would be minimized.

2. Experimental equipment and procedure

The slit-jet flow field is sensitive to disturbances in the flow system. A conventional, or fan-driven, system has several inherent disturbance sources including the 'organ-pipe' resonance of the plenum and the blade passing and associated subharmonic frequencies of the fan. In an effort to avoid these problems, a special, 'collapsing-volume', flow system was constructed for the present investigation; this unit is shown schematically in figure 2.

The cover plate is hinged at the forward end of the chamber and counterbalance weights were added to provide a net torque to lift the plate. The aft endplate of the chamber is formed from a circular arc segment. Hence, as the cover plate is driven

† The lengthscale $2w$ was used by Beavers & Wilson for the Reynolds number. Hence their data extend to $U_0 2w/\nu = 3000$.

‡ The primary thrust of their work was the three-dimensionality of the breakdown of the two-dimensional large-scale vortex motions.

§ The recognition of the third instability and the large ($\gtrsim 1500$) Reynolds numbers of the Clark & Kit study are sufficient to explain the otherwise anomalous $S_T = S_T(w)$ behaviour of their study.

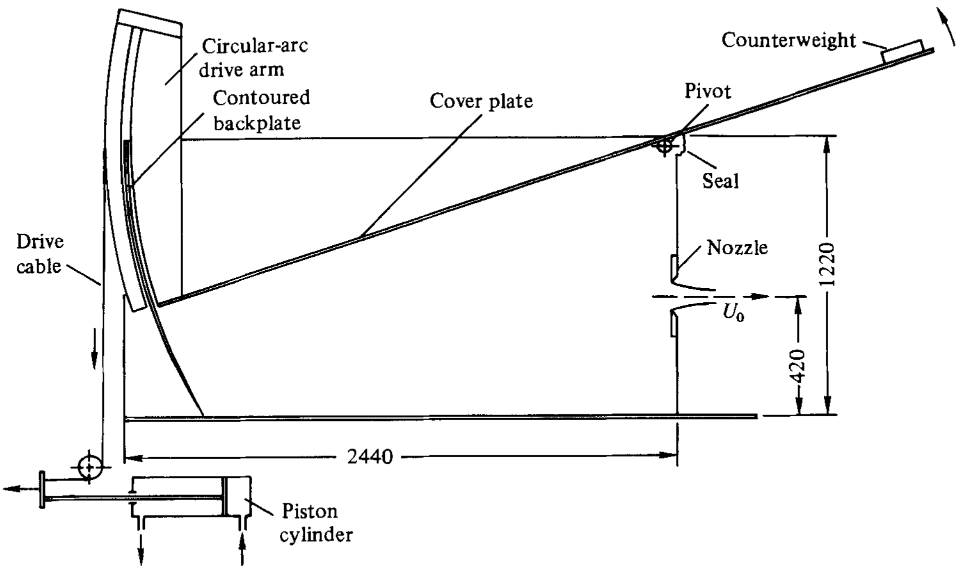

FIGURE 2. Schematic representation of the collapsing-volume flow system.

downward by the extension of the air-cylinder piston, a volume flow is established in proportion to the rate of collapse of the enclosed volume. Specifically,

$$U_0 = \left[\frac{WR^2}{2C_D wL}\right]\frac{d\theta}{dt}, \tag{2}$$

where the discharge coefficient $C_D = 0.611$ (from Valentine 1959), θ = angular position of the cover plate, R = length of the cover plate, and W = width of the enclosed chamber. Equation (2) does not account for a leakage flow, but its form is unchanged if the leakage is constant. This condition was obtained to within the precision of the measurements by adding felt strips as flexible seals at the points of moving contact.

The slit-jet nozzle is formed from four separate elements. This construction allows a wide range of lengths ($L \leqslant 800$ mm) and widths ($8 \text{ mm} \leqslant w \leqslant 32$ mm) to be independently set. However, only one combination, $L = 420$ mm, $w = 16$ mm, aspect ratio = 26.25, was used for the present investigation.

The experimental observations were obtained using three separate hot-wire probes; the placement of the three probes is shown in figure 3. A calibrated wire, located at $(x = w, y = \frac{1}{2}w, z = \frac{1}{4}L)$, was used for the measurement of U_0. (Note that its streamwise location is a small distance beyond the nominal vena contracta: $x \approx 0.7w$.†) The second wire, at $x = w$, $y = 0.19w$, $z = 0.625w$, and the third wire at $X = w + \Delta x$, $y = 0.19w$, $z = -w$, were used to create a cross-correlation $R_{11}(\boldsymbol{x}, \boldsymbol{r}, \tau)$ (see figure 3). The vortex structures of interest were sufficiently regular that the cross-correlation with zero x-separation showed a distinct peak at $\tau = 0$. In addition, the cross-correlation was more easily interpreted than an autocorrelation from a single wire since the small-scale variations in the velocity time histories at the two wires were uncorrelated.

The essential information, from the cross-correlation techniques, is provided by

† The inviscid-flow streamlines converge asymptotically to the width $0.611w$. The nominal vena contract is herein defined as the location where $u(x)/U_0 = 0.98$.

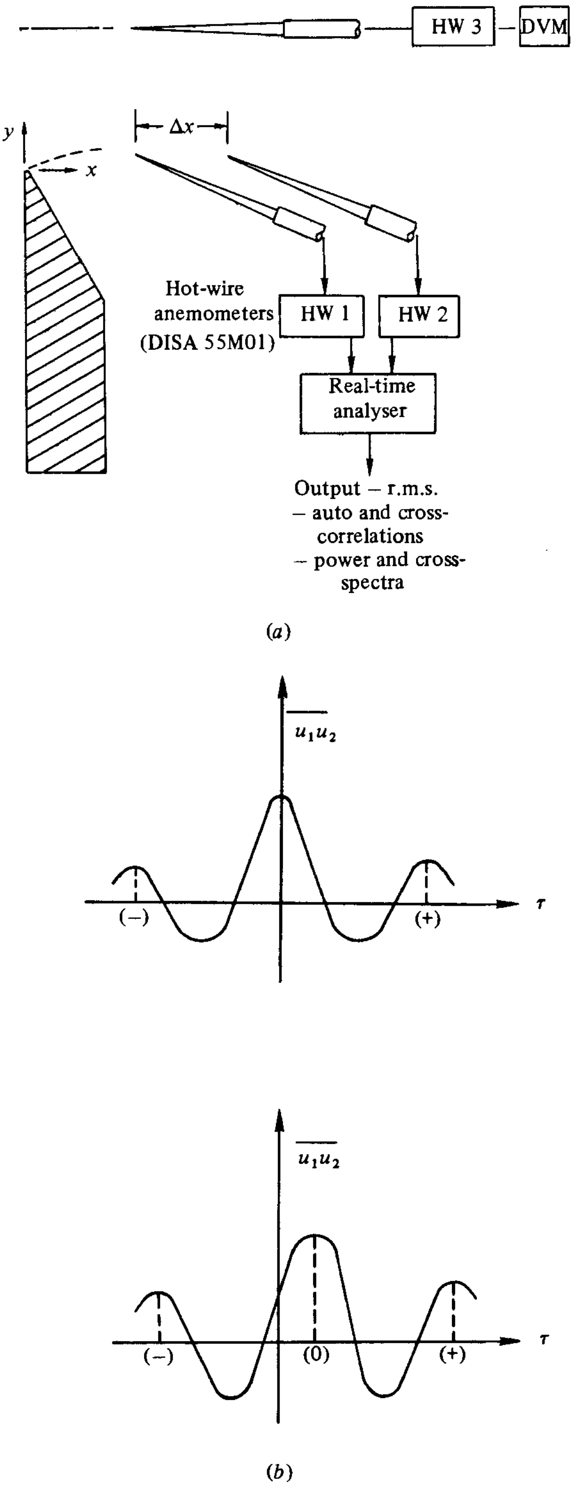

FIGURE 3. Data acquisition equipment and techniques. (*a*) Schematic representation of the flow field and the instrumentation. (*b*) Cross-correlation technique. $\tau_0 = 0$ for $\Delta x = 0$; $\tau_- = 0$ for $\Delta x = \lambda$ (λ wavelength between adjacent structures).

$t_{-}(\Delta x)$, where Δx is the streamwise displacement between the two wires and τ_{-} is the τ-value of the first negative peak (see figure 3). For a zero value of Δx, τ_{-} provides a direct measure of the frequency, since $(\tau_{-})^{-1}$ is the repetition rate at which the vortex structures pass a fixed location. From this initial condition, $\tau_{-} \to 0$ as Δx increases, the critical point $\tau_{-} = 0$ is reached when $\Delta x = \lambda$, the wavelength between adjacent structures. Since $u_c = \lambda f$, the non-dimensional convection speed may be written as

$$\frac{u_c}{U_0} = \frac{fw}{U_0}\frac{\lambda}{w}, \tag{5}$$

where the first grouping fw/U_0 is expected to be 0.43 after Beavers & Wilson, and u_c/U_0 and λ/w are also expected to have universal values.

3. Results and discussion

The collapsing cover plate provided a constant velocity of sufficient duration to acquire 3 s of data for a given experiment. This elapsed time was sufficient for the passage of nominally 200 large-scale motions. Figure 4 presents a representative cross-correlation function for one experiment. The τ_{-} values, from the similar cross-correlation records of sixteen different experiments at four different Δx-values, are presented in figure 5.

The present measurements $\tau_{-}(\Delta x = 0)$ confirm the Beavers & Wilson value for the Strouhal number $S_T = 0.43$. In addition, the measurements for $\Delta x > 0$ provide the values, $\lambda/w \approx 1.2$ and $u_c/U_0 \approx 0.51$. As is evident from the figure, the availability of a set of measurements is most useful in establishing the linear relationship $\tau_{-} = \tau_{-}(\Delta x)$. A second characteristic, made evident in the figure, is that the well-defined τ_{-} value, at a given Δx-location, can take on a range of values. Apparently, the physical process that controls the spacing of the vortex motions is not 'rigidly' determined by the kinematic scales U_0 and w, although this spacing is well-defined for a given experiment, as shown by figure 4.

A point of agreement with the Beavers & Wilson study is implicitly contained in figure 5; that is, an upper bound appears to exist beyond which the regular vortex street no longer exists. A well-defined $\tau_{-}(\Delta x)$ value was obtained for all Reynolds numbers such that $R_e \leqslant 1600$. Conversely, a regular vortex street was not observed for $R_e > 2500$. The existence of a regular vortex street in the intermediate range was apparently dependent upon the ambient disturbance level associated with any one experimental realization.

The present apparatus could not be used to explore the lower bound of Reynolds numbers for a regular vortex street, $R_e \approx 200$ from Beavers & Wilson, since the air cylinder did not operate smoothly at low speed.

Additional information, regarding the qualitative characteristics of the flow field, is available from observations made with the speed-reference wire (at $x = w, y = \frac{1}{2}w, z = \frac{1}{4}L$). When the flow is dominated by the second instability, the centreline velocity exhibits a 'sinusoidal' oscillation about the mean U_0 value. This oscillation is not observed for the large Reynolds numbers associated with the third instability. The relative magnitude of these oscillations was not recorded.

4. Summary

Three modes of instability in the flow from a slit jet are recognized. The formation of small-scale vortex motions, in the shed boundary-layer fluid, is presumed to occur

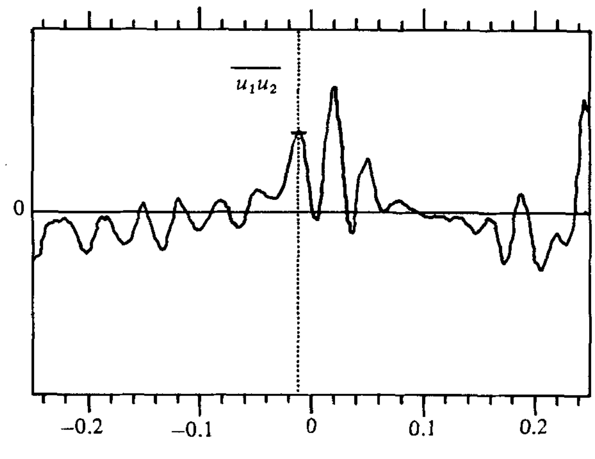

FIGURE 4. A representative cross-correlation $\overline{u_1(t)u_2(t+\tau)}$ between the fixed and the movable probes. $U_0 = 1.28$ m/s, $R_e = 1360, f = 33.1$ Hz, $S_T = 0.44$, ordinate units are arbitrary.

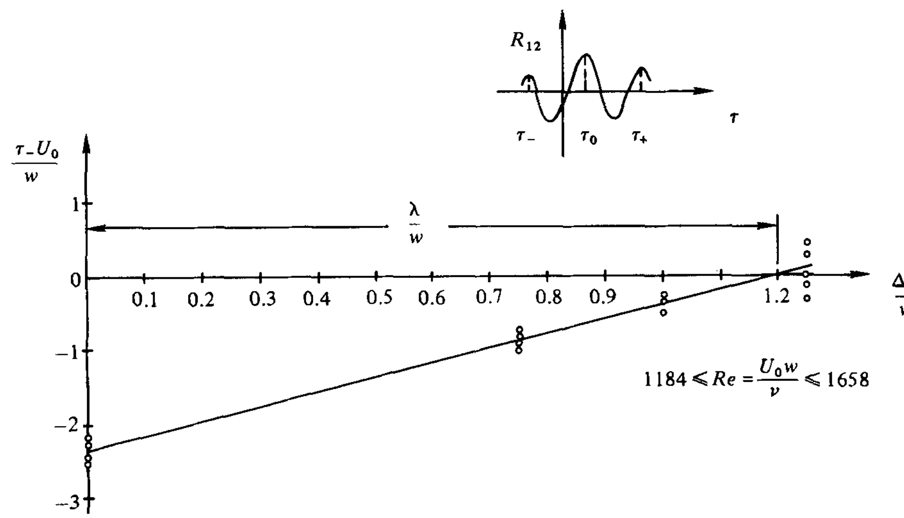

FIGURE 5. The composite results of the cross-correlation measurements.

by analogy to the flow past a bluff object; the appropriate lengthscale for this formation is some measure (e.g. δ^*) the boundary-layer thickness. Hence the associated Strouhal number is Reynolds-number dependent, as shown by Pierce (1964). The second mode may be described as the agglomeration of the unit vortex motions. Beavers & Wilson (1970) report that this process is initiated at $U_0 w/\nu \gtrsim 200$ and is terminated for $U_0 w/\nu \gtrless 1600$ with a nominal Strouhal number of $fw/U_0 \approx 0.43$. The present study corroborates the latter two results and extends these observations to include the wavelength ($\lambda/w \approx 1.2$ and the convection speed ($u_c/U_0 \approx 0.51$). The two studies differ substantially in terms of the apparatus and the measurement techniques; this adds confidence to the interpretation that the observed behaviour is universal. Specifically, the existence of the third instability mode at $U_0 w/\nu \approx 1600$, results in the destruction of the symmetrically placed, larger-scale vortex motions of the second mode. The physical mechanisms that are responsible for the mode-two and mode-three instabilities are the subject of a continuing research programme.

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